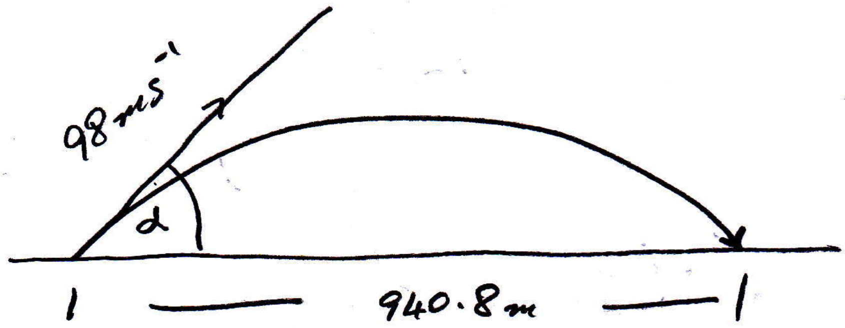
2012 Q3.

(a) (i)



$$S_x = u\cos\alpha\, t$$

$$\Rightarrow \quad 940.8 = 98\cos\alpha\, t$$

$$\Rightarrow \quad t = \frac{940.8}{98\cos\alpha} = \frac{9.6}{\cos\alpha}$$

$$S_y = u\sin\alpha\, t - \tfrac{1}{2}gt^2$$

$$0 = 98\sin\alpha\, t - \tfrac{1}{2}gt^2$$

$$\Rightarrow \quad 0 = t\left(98\sin\alpha - \tfrac{1}{2}gt\right)$$

$$\Rightarrow \quad t = 0 \quad \text{or} \quad 98\sin\alpha - \tfrac{1}{2}gt = 0$$

$$\Rightarrow \quad 98\sin\alpha - \tfrac{1}{2}g\left(\frac{9.6}{\cos\alpha}\right) = 0$$

$$\Rightarrow \quad 98\sin\alpha - \frac{47.04}{\cos\alpha} = 0$$

$$\Rightarrow \quad 98\sin\alpha\cos\alpha - 47.04 = 0$$

(÷ by 49)

$$\Rightarrow \quad 2\sin\alpha\cos\alpha = 0.96$$

$$\Rightarrow \quad \sin(\alpha+\alpha) + \sin(\alpha-\alpha) = 0.96$$

$$\Rightarrow \quad \sin 2\alpha = 0.96$$

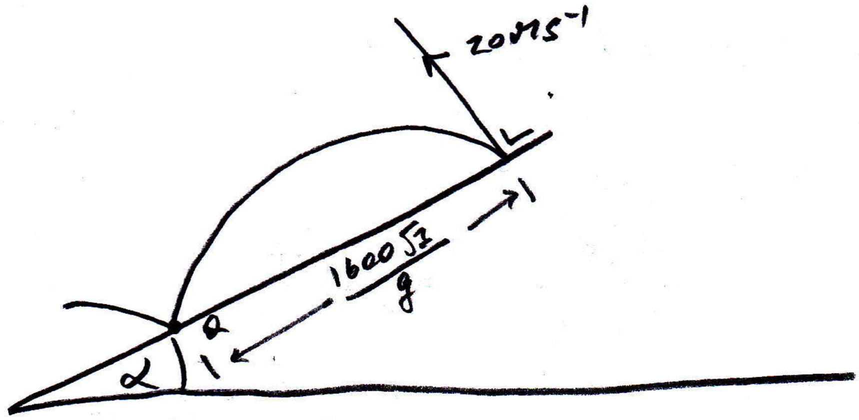
$$\Rightarrow \quad 2\alpha = 73.74^\circ, 106.26 \Rightarrow \alpha = 36.87^\circ, 53.13^\circ$$

(ii) $t_1 = \dfrac{9.6}{\cos 36.87^\circ}$ $t_2 = \dfrac{9.6}{\cos 53.13^\circ}$

$= 12$ $= 16$

$t_2 - t_1 = 4$ sec.

(b)



$$S_y = 20 \sin 90^\circ t - \tfrac{1}{2} g \cos\alpha \, t^2 = 0$$

$$\Rightarrow t\left(20 - \tfrac{1}{2} g \cos\alpha \, t\right) = 0$$

$$\Rightarrow t = 0 \text{ or } 20 - \tfrac{1}{2} g \cos\alpha \, t = 0$$

$$\Rightarrow t = \frac{40}{g \cos\alpha}$$

$$S_x = 20 \cos 90 . t + \tfrac{1}{2} g \sin\alpha \, t^2 = \frac{1600\sqrt{3}}{g}$$

$$\Rightarrow \tfrac{1}{2} g \sin\alpha \left(\frac{40}{g\cos\alpha}\right)^2 = \frac{1600\sqrt{3}}{g}$$

$$\Rightarrow \tfrac{1}{2} g \sin\alpha \frac{1600}{g^2 \cos^2\alpha} = \frac{1600\sqrt{3}}{g}$$

$$\Rightarrow \frac{\sin\alpha}{2\cos^2\alpha} = \sqrt{3}$$

$$\Rightarrow \sin\alpha = 2\sqrt{3}\cos^2\alpha$$

$$\Rightarrow \sin\alpha = 2\sqrt{3}(1-\sin^2\alpha)$$

$$\Rightarrow \sin\alpha = 2\sqrt{3} - 2\sqrt{3}\sin^2\alpha$$

$$\Rightarrow 2\sqrt{3}\sin^2\alpha + \sin\alpha - 2\sqrt{3} = 0$$

Let $y = \sin\alpha$

$$\Rightarrow 2\sqrt{3}\,y^2 + y - 2\sqrt{3} = 0$$

$$\left.\begin{aligned} a &= 2\sqrt{3} \\ b &= 1 \\ c &= -2\sqrt{3} \end{aligned}\right\} \Rightarrow y = \frac{-1 \pm \sqrt{(1)^2 - 4(2\sqrt{3})(-2\sqrt{3})}}{2(2\sqrt{3})}$$

$$= \frac{-1 \pm \sqrt{49}}{4\sqrt{3}}$$

$$= \frac{3}{2\sqrt{3}} \quad \text{or} \quad -\frac{2}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{2} \qquad \times$$

$$\Rightarrow \sin\alpha = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \alpha = 60^\circ \longrightarrow \begin{cases} \sin\alpha = \frac{\sqrt{3}}{2} \\ \cos\alpha = \frac{1}{2} \end{cases}$$

(ii) $$\vec{V}_y = 20\sin 90^\circ - g\cos\alpha\, t$$

$$= 20 - g \cdot \frac{1}{2}\left(\frac{40}{g\cos\alpha}\right)$$

$$= 20 - \frac{20}{1/2}$$

$$= 20 - 40 = -20 \text{ ms}^{-1}$$

$$\vec{V}_x = u \cos 90^\circ + g \sin \alpha \, t$$

$$= 20(0) + g \frac{\sqrt{3}}{2} \left(\frac{40}{g \cos \alpha} \right)$$

$$= \frac{\sqrt{3} \cdot 20}{1/2} = 40\sqrt{3} \text{ ms}^{-1}$$

After the bounce $\vec{V}_x$ is unchanged $= 40\sqrt{3}$ ms^{-1}

and $\vec{V}_y = \frac{1}{2} \times -20 = -10 \text{ ms}^{-1}$

$$|v| = \sqrt{(-10)^2 + (40\sqrt{3})^2}$$

$$= \sqrt{100 + 4800}$$

$$= \sqrt{4900}$$

$$= 70 \text{ m/s}.$$