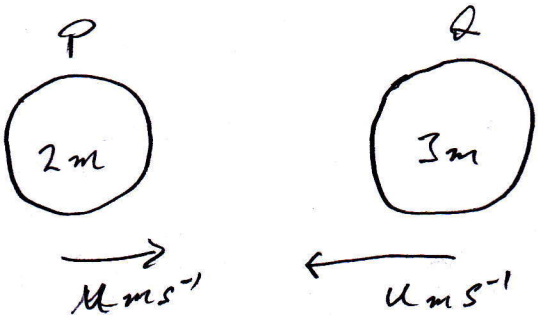
2011 Q5.

(a)(i)



PCM: $2m(u) + 3m(-u) = 2mV_1 + 3mV_2$

$\Rightarrow 2u - 3u = 2V_1 + 3V_2$

$\Rightarrow -u = 2V_1 + 3V_2$ ①

NEL: $V_1 - V_2 = -e(u+u)$

$\Rightarrow V_1 - V_2 = -2ue$ ②

① $-u = 2V_1 + 3V_2$

$\Rightarrow -3V_2 - u = 2V_1 \Rightarrow V_1 = \frac{-3V_2 - u}{2}$

Similarly $V_2 = \frac{-(2V_1 + u)}{3}$

Substituting into ②

$\Rightarrow \frac{-3V_2 - u}{2} - V_2 = -2ue$

$\Rightarrow -3V_2 + u - 2V_2 = -4ue$

$\Rightarrow -5V_2 = -4ue + u \Rightarrow V_2 = \frac{4ue - u}{5}$

$= \frac{u(4e-1)}{5}$

Similarly: $V_1 = \frac{-u(1+6e)}{5}$

Since $0 < e < 1$ V_1 is always negative (forall) $\forall e$

(ii) We need $V_2 > 0$

$$\Rightarrow \frac{u(-1+4e)}{5} > 0$$

$$\Rightarrow u(-1+4e) > 0$$

$$\Rightarrow -u + u4e > 0$$

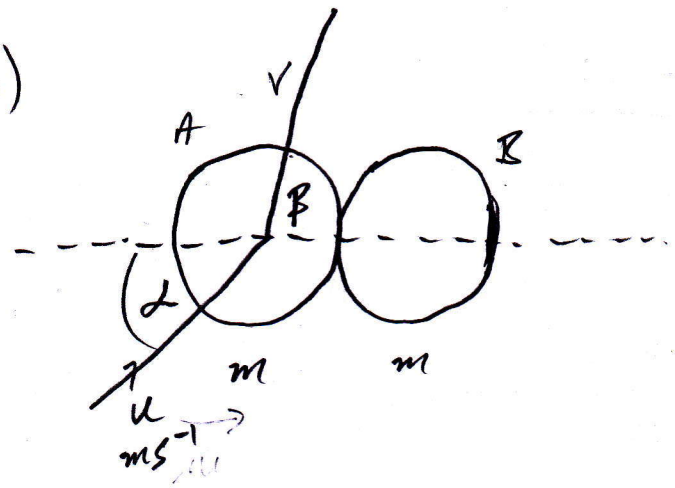
$$\Rightarrow 4ue > u$$

$$\Rightarrow e > \frac{u}{4u}$$

$$\Rightarrow e > \frac{1}{4}$$

$$\Rightarrow \left\{ \frac{1}{4} < e < 1 \right\}$$

(b) (i)



PCM: $m(u\cos\alpha) + m(0) = mV_1 + mV_2$ ①

NEL: $V_1 - V_2 = -e(u\cos\alpha - 0)$ ②

① $u\cos\alpha = V_1 + V_2$ $\left(\Rightarrow V_1 = u\cos\alpha - V_2 \text{ and } V_2 = u\cos\alpha - V_1\right)$

② $-eu\cos\alpha = V_1 - V_2$

Substituting ① into ②

$\Rightarrow -eu\cos\alpha = V_1 - u\cos\alpha + V_1$

$\Rightarrow -eu\cos\alpha + u\cos\alpha = 2V_1$

$\Rightarrow \dfrac{u\cos\alpha(1-e)}{2} = V_1$

Similarly: $\dfrac{u\cos\alpha(1+e)}{2} = V_2$

$\tan\beta = \dfrac{u\sin\alpha}{V_1}$

[Vel. in j direction is unchanged and $V_1 = V\cos\beta$]

$$\Rightarrow \tan\beta = \frac{u\sin\alpha}{\frac{u\cos\alpha(1-e)}{2}}$$

$$= \frac{2u\sin\alpha}{u\cos\alpha(1-e)}$$

$$\tan\beta = \frac{2\tan\alpha}{1-e}$$

$$\Rightarrow \tan\alpha = \frac{(1-e)}{2}\tan\beta$$

Since $\tan\alpha = k\tan\beta$ (given)

$$\Rightarrow k = \frac{1-e}{2}$$

(ii) $I = \frac{7}{8}mu\cos\alpha$

$$\Rightarrow \frac{7}{8}mu\cos\alpha = mV_2 - mu_2$$

$$\Rightarrow \frac{7}{8}mu\cos\alpha = m\left(\frac{u\cos\alpha(1+e)}{2}\right) - m(0)$$

$$\Rightarrow \frac{7}{8}mu\cos\alpha = \frac{mu\cos\alpha(1+e)}{2}$$

$$\Rightarrow \frac{7}{8} = \frac{1+e}{2}$$

$$\Rightarrow 14 = 8+8e$$

$$\Rightarrow \frac{3}{4} = e$$