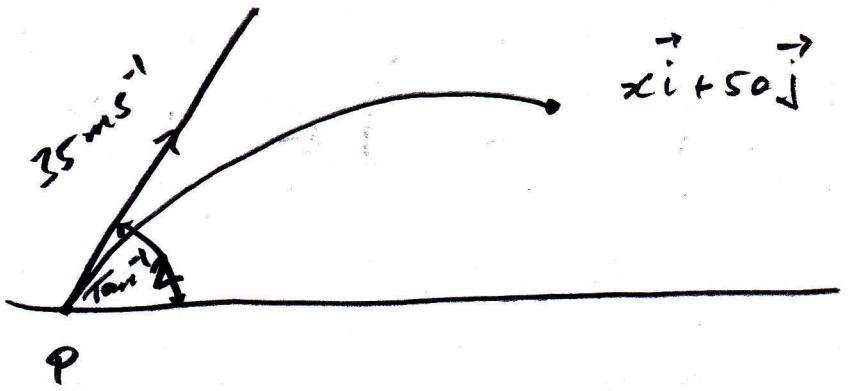
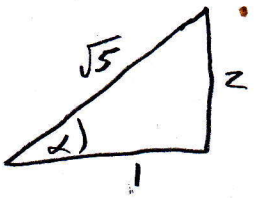
2011 Q3.

(a)



(i)



$s_x = u_x t$

$= 35 \cos\alpha t$

$= 35 \cdot \frac{1}{\sqrt{5}} t \quad \Rightarrow \quad x = 7\sqrt{5}t$

$\Rightarrow \quad t = \frac{x}{7\sqrt{5}}$

$s_y = u_y t - \frac{1}{2} g t^2$

$50 = 35 \sin\alpha t - \frac{1}{2} g t^2$

$\Rightarrow 50 = 35 \cdot \frac{2}{\sqrt{5}} \left(\frac{x}{7\sqrt{5}}\right) - 4.9 \left(\frac{x}{7\sqrt{5}}\right)^2$

$\Rightarrow 50 = 2x - 0.02x^2$

$\Rightarrow 0 = 2x^2 - 200x + 5000$

$\Rightarrow 0 = x^2 - 100x + 2500$

$\Rightarrow 0 = (x - 50)(x - 50)$

$\Rightarrow x = 50$

(ii) $\underline{s_x:}$ $35 \cos\alpha t = 50 \Rightarrow t = \frac{50}{35 \cdot \cos\alpha} = \frac{10}{7\cos\alpha}$

$$35 \sin\alpha t - \frac{1}{2} g t^2 = 50$$

$$\Rightarrow 35 . \sin\alpha \left(\frac{10}{7\cos\alpha}\right) - \frac{1}{2} g \left(\frac{10}{7\cos\alpha}\right)^2 = 50$$

$$\Rightarrow 50 \frac{\sin\alpha}{\cos\alpha} - \frac{10}{\cos^2\alpha} = 50$$

$$\Rightarrow 50 \tan\alpha - 10(1 + \tan^2\alpha) = 50$$

$$\Rightarrow 50 \tan\alpha - 10 - 10\tan^2\alpha = 50$$

$$\Rightarrow \tan^2\alpha - 5\tan\alpha + 6 = 0$$

$$\Rightarrow (\tan\alpha - 2)(\tan\alpha - 3) = 0$$

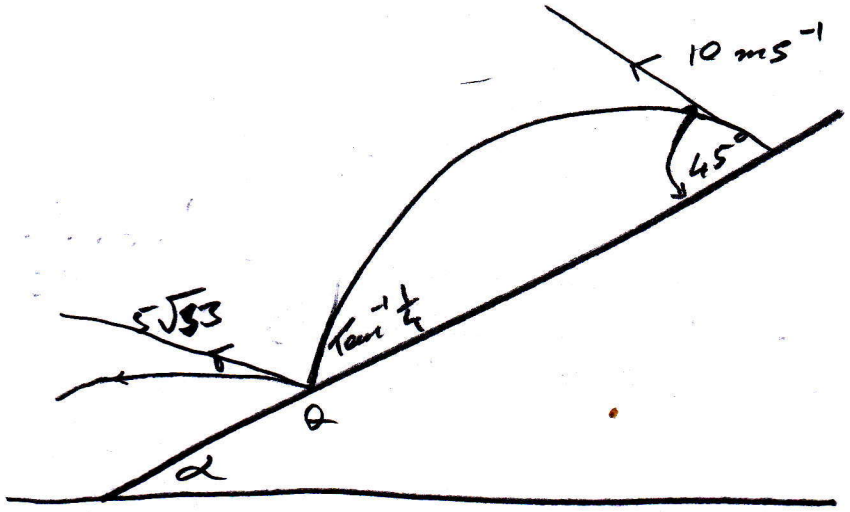
$$\Rightarrow \tan\alpha = 2 \text{ or } 3$$

$\tan^{-1} 3 = 71.6^\circ$ [We were given $\tan^{-1} 2$]

Note:

$\tan^2\theta + 1 = \sec^2\theta$

(b)



(i) $S_y = u \sin 45° t - \frac{1}{2} g \cos\alpha t^2 = 0$

$\Rightarrow t(u \sin 45° - 4.9 \cos\alpha t) = 0$

$\Rightarrow t = 0$ or $10 \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} g \cos\alpha t = 0$

$\Rightarrow t = \frac{10 \cdot 2}{\sqrt{2} g \cos\alpha}$

$\Rightarrow t = \frac{10\sqrt{2}}{g \cos\alpha}$

$V_x = 10 \cos 45 + g \sin\alpha t$

$V_x = \frac{10}{\sqrt{2}} + g \sin\alpha \left(\frac{10\sqrt{2}}{g \cos\alpha}\right)$

$V_x = 5\sqrt{2} + 10\sqrt{2} \tan\alpha$

$V_y = 10 \sin 45 - g \cos\alpha \left(\frac{10\sqrt{2}}{g \cos\alpha}\right)$

$= 5\sqrt{2} - 10\sqrt{2}$

$= -5\sqrt{2}$

$$\tan\theta = \frac{-V_y}{V_x}$$

$$\Rightarrow \frac{1}{4} = \frac{5\sqrt{2}}{5\sqrt{2} + 10\sqrt{2}\tan\alpha}$$

$$\Rightarrow 5\sqrt{2} + 10\sqrt{2}\tan\alpha = 20\sqrt{2}$$

$$\Rightarrow \tan\alpha = \frac{15\sqrt{2}}{10\sqrt{2}}$$

$$\Rightarrow \tan\alpha = \frac{3}{2}$$

$$\Rightarrow \alpha = 56.3^\circ$$

(ii) $V_y = -5\sqrt{2}$ $\quad V_x = 5\sqrt{2} + 10\sqrt{2}\tan\alpha$

$$= 5\sqrt{2} + 10\sqrt{2}\cdot\frac{3}{2}$$

$$= 20\sqrt{2}$$

$$\Rightarrow 5\sqrt{33} = \sqrt{(-5e\sqrt{2})^2 + (20\sqrt{2})^2}$$

$$\Rightarrow 5\sqrt{33} = \sqrt{50e^2 + 800}$$

$$\Rightarrow 825 = 50e^2 + 800$$

$$\Rightarrow 25 = 50e^2$$

$$\Rightarrow \frac{1}{2} = e^2$$

$$\Rightarrow \frac{1}{\sqrt{2}} = e$$