2012 Q10.

(a) (i)

$$\frac{d\theta}{dt} = -k\theta$$

$$\Rightarrow \frac{1}{\theta}\,d\theta = -k\,dt$$

$$\Rightarrow \int_{60}^{40} \frac{1}{\theta}\,d\theta = -k\int_{0}^{10} dt$$

$$\Rightarrow \Big[\ln\theta\Big]_{60}^{40} = -k\Big[t\Big]_{0}^{10}$$

$$\Rightarrow \ln 40 - \ln 60 = -10k$$

$$\Rightarrow k = 0.0405$$

(ii)

$$\Big[\ln\theta\Big]_{40}^{\theta-20} = -k\Big[t\Big]_{0}^{15}$$

$$\Rightarrow \ln(\theta-20) - \ln 40 = -15k$$

$$\Rightarrow \ln\left(\frac{\theta-20}{40}\right) = -15(0.0405)$$

$$\Rightarrow e^{-0.6075} = \frac{\theta-20}{40}$$

$$\Rightarrow 40e^{-0.6075} = \theta - 20$$

$$\Rightarrow 40e^{-0.6075} + 20 = \theta$$

$$\Rightarrow 41.79^{\circ} = \theta$$

(b) (i) $F = ma$

$$\Rightarrow \quad ma = mkV^2$$

$$\Rightarrow \quad a = kV^2$$

$$\Rightarrow \quad \frac{dV}{dt} = -kV^2 \quad \left(\text{since the particle is decelerating}\right)$$

$$\Rightarrow \quad \frac{1}{V^2} = -k\,dt$$

$$\Rightarrow \quad \int_{1000}^{10} \frac{1}{V^2} = -k\int_0^{0.01} dt$$

$$\Rightarrow \quad \left[-\frac{1}{V}\right]_{1000}^{10} = -k\left[t\right]_0^{0.01}$$

$$\Rightarrow \quad -\frac{1}{10} + \frac{1}{1000} = -k(0.01)$$

$$\Rightarrow \quad \frac{99}{10} = k$$

(ii) Since we need distance we use $a = V\frac{dV}{dx}$

$$V\frac{dV}{dx} = -kV^2 \quad \Rightarrow \quad V\frac{dV}{dx} = -\frac{99}{10}V^2$$

$$\Rightarrow \quad \frac{1}{V}\,dV = -\frac{99}{10}\,dx$$

$$\Rightarrow \quad \int_{1000}^{10} \frac{1}{V}\,dV = -\frac{99}{10}\int_0^{x} dx$$

$$\Rightarrow \left[\ln V\right]_{1000}^{10} = -\frac{99}{10}\left[x\right]_{0}^{x}$$

$$\Rightarrow \ln 10 - \ln 1000 = -\frac{99}{10}x$$

$$\Rightarrow \ln \frac{10}{1000} = -\frac{99}{10}x$$

$$\Rightarrow \ln 100^{-1} = -\frac{99}{10}x$$

$$\Rightarrow -1\ln 100 = -\frac{99}{10}x$$

$$\Rightarrow \frac{10}{99}\ln 100 = x$$